10.2 Find Arc Measures

Key Vocabulary
- **central angle**
- **minor arc**
- **major arc**
- **semicircle**
- **measure**
  minor arc, major arc
- **congruent circles**
- **congruent arcs**

A **central angle** of a circle is an angle whose vertex is the center of the circle. In the diagram, $\angle ACB$ is a central angle of $\odot C$.

If $m\angle ACB$ is less than $180°$, then the points on $\odot C$ that lie in the interior of $\angle ACB$ form a **minor arc** with endpoints A and B. The points on $\odot C$ that do not lie on minor arc $\overarc{AB}$ form a **major arc** with endpoints A and B. A **semicircle** is an arc with endpoints that are the endpoints of a diameter.
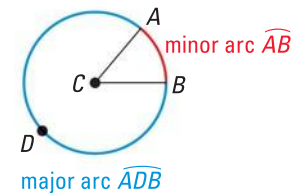
NAMING ARCS Minor arcs are named by their endpoints. The minor arc associated with $\angle ACB$ is named $\overarc{AB}$. Major arcs and semicircles are named by their endpoints and a point on the arc. The major arc associated with $\angle ACB$ can be named $\overarc{ADB}$.
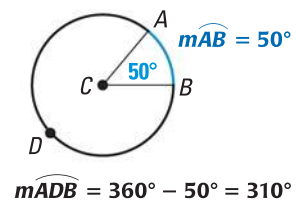

KEY CONCEPT   *For Your Notebook*

Measuring Arcs

The **measure of a minor arc** is the measure of its central angle. The expression $m\overarc{AB}$ is read as "the measure of arc AB."
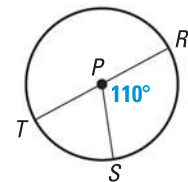
The measure of the entire circle is $360°$. The **measure of a major arc** is the difference between $360°$ and the measure of the related minor arc. The measure of a semicircle is $180°$.

$m\overarc{AB} = 50°$

$m\overarc{ADB} = 360° - 50° = 310°$

EXAMPLE 1   Find measures of arcs

Find the measure of each arc of $\odot P$, where $\overline{RT}$ is a diameter.

   a. $\overarc{RS}$ **b.** $\overarc{RTS}$ **c.** $\overarc{RST}$

Solution

  a. $\overarc{RS}$ is a minor arc, so $m\overarc{RS} = m\angle RPS = 110°$.

  b. $\overarc{RTS}$ is a major arc, so $m\overarc{RTS} = 360° - 110° = 250°$.

  c. $\overline{RT}$ is a diameter, so $\overarc{RST}$ is a semicircle, and $m\overarc{RST} = 180°$.
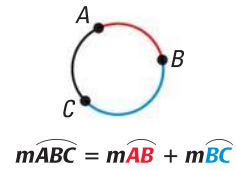
ADJACENT ARCS Two arcs of the same circle are *adjacent* if they have a common endpoint. You can add the measures of two adjacent arcs.

**POSTULATE**   *For Your Notebook*

POSTULATE 23 **Arc Addition Postulate**

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$$

EXAMPLE 2 **Find measures of arcs**

SURVEY A recent survey asked teenagers if they would rather meet a famous musician, athlete, actor, inventor, or other person. The results are shown in the circle graph. Find the indicated arc measures.

Whom Would You Rather Meet?

Musician 108° C 83° Athlete
D
B 29° 61°
Inventor A 79° E Other
Actor
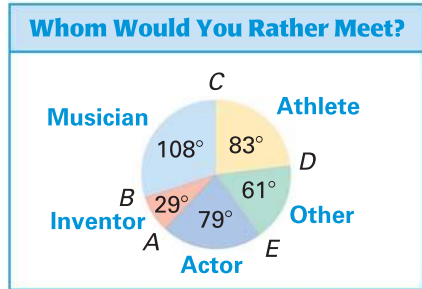
a. $m\widehat{AC}$ b. $m\widehat{ACD}$

c. $m\widehat{ADC}$ d. $m\widehat{EBD}$

Solution

ARC MEASURES

The measure of a minor arc is less than 180°. The measure of a major arc is greater than 180°.

a. $m\widehat{AC} = m\widehat{AB} + m\widehat{BC}$

$= 29° + 108°$

$= 137°$

b. $m\widehat{ACD} = m\widehat{AC} + m\widehat{CD}$

$= 137° + 83°$

$= 220°$

c. $m\widehat{ADC} = 360° - m\widehat{AC}$

$= 360° - 137°$

$= 223°$

d. $m\widehat{EBD} = 360° - m\widehat{ED}$

$= 360° - 61°$

$= 299°$

✓ **GUIDED PRACTICE** **for Examples 1 and 2**

Identify the given arc as a *major arc, minor arc,* or *semicircle,* and find the measure of the arc.

1. $\widehat{TQ}$ 2. $\widehat{QRT}$ 3. $\widehat{TQR}$

4. $\widehat{QS}$ 5. $\widehat{TS}$ 6. $\widehat{RST}$

T 120° Q
80° 60°
S R
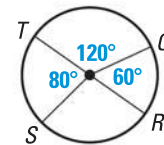
CONGRUENT CIRCLES AND ARCS Two circles are **congruent circles** if they have the same radius. Two arcs are **congruent arcs** if they have the same measure and they are arcs of the same circle or of congruent circles. If $\odot C$ is congruent to $\odot D$, then you can write $\odot C \cong \odot D$.

EXAMPLE 3 Identify congruent arcs

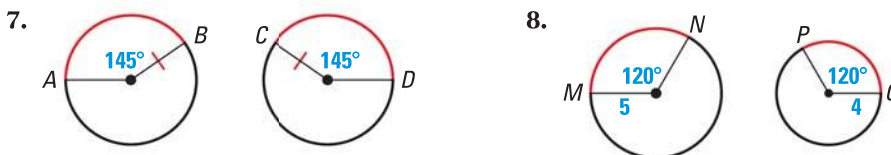**Tell whether the red arcs are congruent. Explain why or why not.**

a.

b.

c.
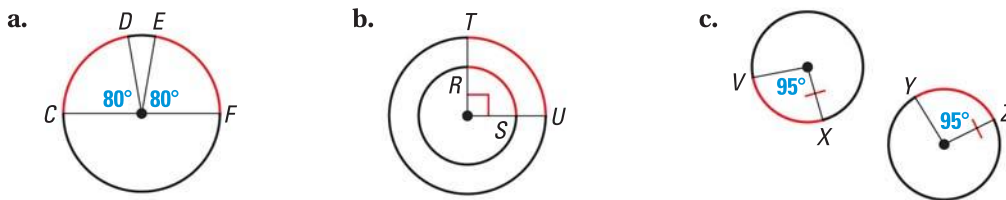
Solution

a. $\overset{\frown}{CD} \cong \overset{\frown}{EF}$ because they are in the same circle and $m\overset{\frown}{CD} = m\overset{\frown}{EF}$.

b. $\overset{\frown}{RS}$ and $\overset{\frown}{TU}$ have the same measure, but are not congruent because they are arcs of circles that are not congruent.

c. $\overset{\frown}{VX} \cong \overset{\frown}{YZ}$ because they are in congruent circles and $m\overset{\frown}{VX} = m\overset{\frown}{YZ}$.

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✔ **GUIDED PRACTICE** for Example 3

Tell whether the red arcs are congruent. *Explain* why or why not.

7.
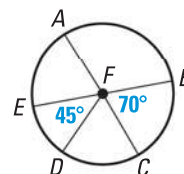
8.

10.2 EXERCISES

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: If $\angle ACB$ and $\angle DCE$ are congruent central angles of $\odot C$, then $\overset{\frown}{AB}$ and $\overset{\frown}{DE}$ are __?__.

2. ★ **WRITING** What do you need to know about two circles to show that they are congruent? *Explain*.

MEASURING ARCS $\overline{AC}$ and $\overline{BE}$ are diameters of $\odot F$. Determine whether the arc is a *minor arc*, a *major arc*, or a *semicircle* of $\odot F$. Then find the measure of the arc.
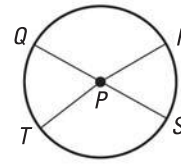
3. $\overset{\frown}{BC}$

4. $\overset{\frown}{DC}$

5. $\overset{\frown}{DB}$

6. $\overset{\frown}{AE}$

7. $\overset{\frown}{AD}$

8. $\overset{\frown}{ABC}$

9. $\overset{\frown}{ACD}$

10. $\overset{\frown}{EAC}$

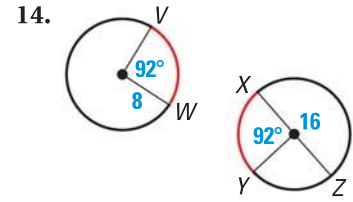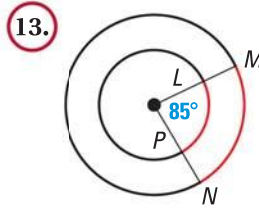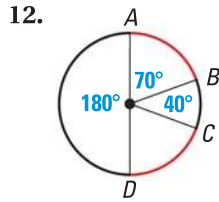**11.** ★ **MULTIPLE CHOICE** In the diagram, $\overline{QS}$ is a diameter of $\odot P$. Which arc represents a semicircle?

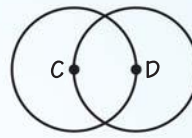Ⓐ $\overset{\frown}{QR}$ Ⓑ $\overset{\frown}{RQT}$

Ⓒ $\overset{\frown}{QRS}$ Ⓓ $\overset{\frown}{QRT}$

CONGRUENT ARCS Tell whether the red arcs are congruent. *Explain* why or why not.

12.   **13.**   **14.**

15. **ERROR ANALYSIS** *Explain* what is wrong with the statement.
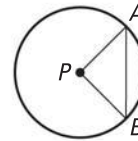
> You cannot tell if $\odot C \cong \odot D$ because the radii are not given.

16. **ARCS** Two diameters of $\odot P$ are $\overline{AB}$ and $\overline{CD}$. If $m\overset{\frown}{AD} = 20°$, find $m\overset{\frown}{ACD}$ and $m\overset{\frown}{AC}$.

17. ★ **MULTIPLE CHOICE** $\odot P$ has a radius of 3 and $\overset{\frown}{AB}$ has a measure of 90°. What is the length of $\overline{AB}$?
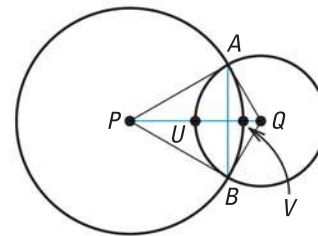
Ⓐ $3\sqrt{2}$ Ⓑ $3\sqrt{3}$

Ⓒ 6 Ⓓ 9

18. ★ **SHORT RESPONSE** On $\odot C$, $m\overset{\frown}{EF} = 100°$, $m\overset{\frown}{FG} = 120°$, and $m\overset{\frown}{EFG} = 220°$. If H is on $\odot C$ so that $m\overset{\frown}{GH} = 150°$, *explain* why H must be on $\overset{\frown}{EF}$.

19. **REASONING** In $\odot R$, $m\overset{\frown}{AB} = 60°$, $m\overset{\frown}{BC} = 25°$, $m\overset{\frown}{CD} = 70°$, and $m\overset{\frown}{DE} = 20°$. Find two possible values for $m\overset{\frown}{AE}$.

20. **CHALLENGE** In the diagram shown, $\overline{PQ} \perp \overline{AB}$, $\overline{QA}$ is tangent to $\odot P$, and $m\overset{\frown}{AVB} = 60°$. What is $m\overset{\frown}{AUB}$?

21. **CHALLENGE** In the coordinate plane shown, C is at the origin. Find the following arc measures on $\odot C$.

   a. $m\overset{\frown}{BD}$

   b. $m\overset{\frown}{AD}$

   c. $m\overset{\frown}{AB}$

⭕ = **WORKED-OUT SOLUTIONS**
on p. WS1

★ = **STANDARDIZED TEST PRACTICE**
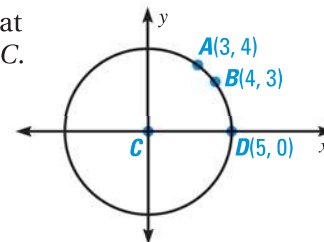
22. BRIDGES The deck of a bascule bridge creates an arc when it is moved from the closed position to the open position. Find the measure of the arc.

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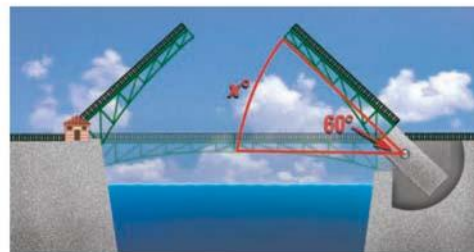
(23.) DARTS On a regulation dartboard, the outermost circle is divided into twenty congruent sections. What is the measure of each arc in this circle?

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24. ★ EXTENDED RESPONSE A surveillance camera is mounted on a corner of a building. It rotates clockwise and counterclockwise continuously between Wall A and Wall B at a rate of 10° per minute.

   a. What is the measure of the arc surveyed by the camera?

   b. How long does it take the camera to survey the entire area once?

   c. If the camera is at an angle of 85° from Wall B while rotating counterclockwise, how long will it take for the camera to return to that same position?

   d. The camera is rotating counterclockwise and is 50° from Wall A. Find the location of the camera after 15 minutes.
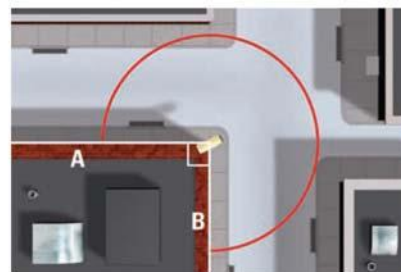
25. CHALLENGE A clock with hour and minute hands is set to 1:00 P.M.

   a. After 20 minutes, what will be the measure of the minor arc formed by the hour and minute hands?

   b. At what time before 2:00 P.M., to the nearest minute, will the hour and minute hands form a diameter?
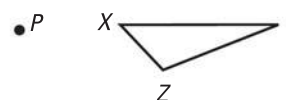
MIXED REVIEW

Determine if the lines with the given equations are parallel. *(p. 180)*

26. $y = 5x + 2, y = 5(1 - x)$ **27.** $2y + 2x = 5, y = 4 - x$

28. Trace $\triangle XYZ$ and point P. Draw a counterclockwise rotation of $\triangle XYZ$ 145° about P. *(p. 598)*

Find the product. *(p. 641)*

29. $(x + 2)(x + 3)$ **30.** $(2y - 5)(y + 7)$ **31.** $(x + 6)(x - 6)$

32. $(z - 3)^2$ **33.** $(3x + 7)(5x + 4)$ **34.** $(z - 1)(z - 4)$